

Mathematics: High School

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Cross Referencing Codes

- NJ = New Jersey Core Standards. The number following “NJ” indicates the numerical designation for a particular standard.
- ACTFL = Performance Guidelines from American Council on the Teaching of Foreign Languages.
- CK = Content Knowledge (cf. Appendix A)
The letters “FL” following the “CK”, indicate the subject area to which the reference is referring. The number following the letters indicates the numerical designation for a particular standard. Thus, CK FL 3 translates Foreign Language Standard # 3 in Content Knowledge.

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High School Mathematics
CH Mathematics Standards 3:11 – 3:20
(Click on a standard to view its list of benchmarks)

CH 3: 11	The student understands the nature and general use of mathematics.
CH 3: 12	The student understands and applies basic and advanced properties of the concepts of numbers.
CH 3: 13	The student develops an understanding of and uses geometry and measurement concepts to describe and analyze phenomena.
CH 3: 14	The student understands and applies basic and advanced properties of functions of algebra.
CH 3: 15	The student understands and applies basic and advanced concepts of statistics and data analysis.
CH 3: 16	The student understands and applies basic and advanced concepts of probability.
CH 3: 17	The student develops and demonstrates the ability to pose and solve mathematical problems, communicating mathematically through written, oral, symbolic and visual forms of expression.
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High School Mathematics (continued)

CH Mathematics Standards 3:11 – 3:20
(Click on a standard to view its list of benchmarks)

CH 3: 18	The student understands, selects, and applies various methods of performing numerical operations.
CH 3: 19	The student communicates mathematically through written, oral, symbolic, and visual forms of expression.
CH 3: 20	The student applies mathematical knowledge and skills to relate to real-world situations.
CH 3: 21	The student presents evidence that demonstrates understanding of differential and integral calculus.

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High School Mathematics

<p>Standard CH 3: 11</p> <p>The student understands the nature and general use of mathematics.</p> <p>(NJ 4.3; NCTM; CK M9)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">□ Understands the manner in which mathematics has been helpful in practical ways for many centuries□ Understands that in mathematics, as in other sciences, simplicity is one of the highest values; some mathematicians try to identify the smallest set of rules from which many other propositions can be logically derived□ Understands that theories in mathematics are greatly influenced by practical issues; real-world problems sometimes result in new mathematical theories, and pure mathematical theories sometimes have highly practical applications□ Understands that science and mathematics operate under common principles: belief in order, ideas of honesty and openness, the importance of review by colleagues, and the importance of imagination□ Understands that mathematics provides a precise system to describe objects, events, and relationships, and to construct logical arguments□ Understands that technological progress has opened and will continue to open new doors to mathematics□ Understands that mathematics often stimulates innovations in science and technology
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High School Mathematics

<p>Standard CH 3: 12</p> <p>The student understands and applies basic and advanced properties of the concepts of numbers.</p> <p>(NS M1; NJ 4.1, NJ 4.5, NJ 4.6, NJ 4.8; NCTM; CK M2, CK M8)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">□ Adds, subtracts, multiplies, divides, and simplifies rational expressions and radical expressions□ Adds, subtracts, multiplies, divides, and simplifies radical expressions containing positive rational numbers]□ Demonstrates understanding of the properties and basic theorems of roots and exponents□ Understands and applies operations such as opposite, raising to a power, reciprocal, and taking a logarithm□ Understands and uses number systems: natural, integer, rational, real, and complex□ Uses numbers in decimal or fraction form and in scientific notation□ Demonstrates an ability to perform all necessary and appropriate graphing activities□ Demonstrates the ability to compare numbers through the use of ratios, differences, percents, proportions, and order relations <p>*Continued on next page*</p>
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High School Mathematics

<p>Standard CH 3: 12 (Continued)</p> <p>The student understands and applies basic and advanced properties of the concepts of numbers.</p> <p>(NS M1; NJ 4.1, NJ 4.5, NJ 4.6, NJ 4.8; NCTM; CK M2, CK M8)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">□ Compares numbers using proportional changes and carries out proportional reasoning in cases such as part-whole reasoning, expansions and contractions□ Uses proportions, percents and multiplicative factors to demonstrate an understanding of all types of measurement□ Understands counting procedures and reasoning (e.g., use of the Addition Counting Principle to find the number of ways of arranging objects in a set, the use of permutations and combinations to solve counting problems)□ Uses number theory concepts (divisibility and remainders, factors, multiples, prime, relatively prime) in solving problems□ Uses scientific calculators and computers effectively and efficiently in applying mathematical concepts and principles to various types of problems□ Identifies and computes key number patterns such as those involving multiples, squares or cubes□ Uses written statements with frequency to summarize and explain number and operation concepts
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High School Mathematics

Standard CH 3: 13	Benchmarks (High School)
<p>The student develops an understanding of and uses geometry and measurement concepts to describe and analyze phenomena.</p> <p>(NS M2; NJ 4.1, NJ 4.2, NJ 4.4, NJ 4.5, NJ 4.7, NJ 4.9; NCTM; CK M1, CK M4, CK M5)</p>	<ul style="list-style-type: none">□ Demonstrates understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning□ Writes geometric proofs, including proofs by contradiction□ Proves basic theorems involving congruence and similarity□ Analyzes properties of two and three dimensional shapes, including circles, polygons, cubes, pyramids, cones, spheres, and cylinders□ Works with a variety of geometric measures such as length, area, volume and angle, and with non-geometric measures such as weight and time□ Computes areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids□ Finds and uses measures of sides, and of interior and exterior angles of triangles and polygons to classify figures and solve problems□ Uses geometric language to describe a variety of figures, objects, paths, and regions in space, including intersections and cross sections of three dimensional figures□ Knows, derives and solves problems involving the perimeter, circumference, volume, lateral area, and surface area of common geometric figures□ Uses the Pythagorean Theorem and its converse and properties of special right triangles (e.g., 30, 60, 90 degree triangle) to solve mathematical and real-world problems <p>*Continued on next page*</p>

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High School Mathematics

<p>Standard CH 3: 13 (Continued)</p> <p>The student develops an understanding of and uses geometry and measurement concepts to describe and analyze phenomena.</p> <p>(NS M2; NJ 4.1, NJ 4.2, NJ 4.4, NJ 4.5, NJ 4.7, NJ 4.9; NCTM; CK M1, CK M4, CK M5)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">□ Knows the definitions of and successfully applies the three basic trigonometric functions□ Knows the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections□ Uses synthetic (i.e. pictorial) representations and analytic (i.e. coordinate) methods to solve problems involving symmetry and transformations of figures (e.g., problems involving distance, midpoint and slope (rise over run; determination of symmetry with respect to a point or line)□ Analyzes geometric patterns, including sequences of growing shapes□ Solves problems involving rate as measure (e.g., velocity, acceleration)□ Selects and uses an appropriate direct or indirect method of measurement in a given situation, including quotient measures, units, square units, cubic units; and product measures, such as person-hours□ Solves problems involving units of measurement, both standard and metric, and converts answers to a larger or smaller unit within the same system and in a different system□ Analyzes a variety of problem solving techniques in the use of maps and other diagrams□ Explores systems such as navigational, polar, spherical, and complex□ Uses calculators, computers, manipulatives, and other mathematical tools to enhance mathematical thinking, understanding, and power
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High School Mathematics

<p>Standard CH 3: 14</p> <p>The student understands and applies basic and advanced properties of functions of algebra.</p> <p>(NS M3; NJ 4.1, NJ 4.2, NJ 4.5, NJ 4.6, NJ 4.8, NJ 4.11, NJ 4.13; NCTM; CK M8)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">□ Understands various representations (e.g., tables, graphs, verbal descriptions, algebraic expressions, Venn diagram) of patterns and functions and the relationships among these representations□ Understands and applies the basic concept of a function (i.e., functions describe how changes in one quantity or variable result in changes in another)□ Models and interprets formulas in terms of given situations□ Understands and uses appropriate terminology and notations used to define functions and their properties (e.g., domain, range, function composition, inverses)□ Understands and applies the general properties and characteristics of many types of functions (e.g., direct and inverse variations, general polynomial, radical, step, exponential, logarithmic, sinusoidal)□ Graphs functions and determines the maxima, minima, and zeros of the function□ Demonstrates the ability to utilize concepts involving shape and evaluation□ Solves equations and inequalities involving absolute value□ Solves systems of linear equations and inequalities (in two or three variables) by substitution, with graphics, or with matrices□ Uses equations to represent curves <p>*Continued on next page*</p>
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High School Mathematics

<p>Standard CH 3: 14 (Continued)</p> <p>The student understands and applies basic and advanced properties of functions of algebra.</p> <p>(NS M3; NJ 4.1, NJ 4.2, NJ 4.5, NJ 4.6, NJ 4.8, NJ 4.11, NJ 4.13; NCTM; CK M8)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">□ Demonstrates understanding of basic concepts (e.g., roots) and applications (e.g., determining cost, revenue, and profit situations) of polynomial equations□ Expresses rates numerically, symbolically and graphically, using a variety of models□ Uses linear functions to represent proportional relationships□ Demonstrates the ability to perform mathematical functions such as using arithmetical and geometric sequences, making predictions by interpolation or extrapolation, comparing/contrasting properties of numbers and number systems□ Uses and manipulates expressions, equations, inequalities and matrices to represent situations that involve variable quantities and translates among these representations□ Understands the characteristics and uses of basic trigonometric functions (e.g., the sine and cosine functions as models of periodic real-world phenomena)□ Demonstrates knowledge of how real and complex numbers are related both arithmetically and graphically through use of formulas, graphs, and tables□ Calculates slope and interprets it as the amount of one quantity per unit amount of another□ Develops fluency operating on polynomials, vectors, and matrices□ Uses calculators, computers, manipulatives, and other mathematical tools to enhance mathematical thinking, understanding, and power
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High School Mathematics

<p>Standard CH 3: 15</p> <p>The student understands and applies basic and advanced concepts of statistics and data analysis.</p> <p>(NS M4; NJ 4.1, NJ 4.2, NJ 4.5, NJ 4.12, NJ 4.16; NCTM; CK M6)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">□ Selects, analyzes and organizes a given set of data□ Selects and applies the best method of representing and describing a set of data (e.g., scatter plot, line and circle graphs, two-way tables, histograms)□ Demonstrates understanding of measures of central tendency and variability (e.g., standard deviation, range, quartile deviation) and their applications to specific situations□ Demonstrates understanding of the concept of correlation (e.g., the difference between a “true” correlation and a “believable” correlation; when two variables are correlated)□ Demonstrates understanding of the different methods of curve-fitting (e.g., median-fit line, regression line) and various applications (e.g., making predictions) of these methods□ Creates computer-generated regression lines and correlation coefficients <p>*Continued on next page*</p>
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High School Mathematics

<p>Standard CH 3: 15 (Continued)</p> <p>The student understands and applies basic and advanced concepts of statistics and data analysis.</p> <p>(NS M4; NJ 4.1, NJ 4.2, NJ 4.5, NJ 4.12, NJ 4.16; NCTM; CK M6)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">□ Demonstrates an understanding of how outliers may affect various representations of data (e.g., a regression line might be strongly influenced by a few aberrant points, whereas the scatter plot for the same data might suggest that the aberrant points represent mistakes)□ Demonstrates understanding of sampling distributions, the central limit theorem, and confidence intervals□ Uses sampling techniques to draw inferences about a variety of real-life situations□ Analyzes data and hypothesizes as to possible solutions to real-world problems□ Understands that making an inference about a population from a sample always involves uncertainty, and the role of statistics is to estimate the size of that uncertainty□ Understands how the reader's bias, measurement error, and display distortion can affect the interpretation of data□ Understands how concepts of representativeness, randomness, and bias in sampling can affect experimental outcomes and statistical interpretations
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High School Mathematics

<p>Standard CH 3: 16</p> <p>The student understands and applies basic and advanced concepts of probability.</p> <p>(NS M4; NJ 4.1, NJ 4.2, NJ 4.5, NJ 4.12; NCTM; CK M7)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">□ Demonstrates understanding of the relationship between the numerical expression of a probability (e.g., fraction, percentage, odds) and the events that produce these numbers□ Demonstrates understanding of the concept of a random variable□ Demonstrates understanding of the concepts of independent and dependent events, and how they are related to compound events and conditional probability□ Uses a variety of experimental, simulation and theoretical methods (e.g., counting procedures, trees, formulas for permutations and combinations, Monte Carlo simulations, statistical experiments) determine probabilities□ Creates and uses models of probabilistic situations and understands the role of assumption in this process□ Demonstrates an understanding of the differences among experimental, simulation, and theoretical probability techniques, and the advantages and disadvantages of each□ Demonstrates an understanding of the properties of the normal curve (i.e., the graph used to approximate the distribution of data for many real-world phenomena), and how the normal curve can be used to answer questions about sets of data□ Understands the concept of discrete probability distribution
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High School Mathematics

Standard CH 3: 17	Benchmarks (High School)
The student develops and demonstrates the ability to pose and solve mathematical problems, communicating mathematically through written, oral, symbolic, and visual forms of expression.	<ul style="list-style-type: none">□ Identifies a problem situation, lists pertinent information as a part of the problem-solving process through use of discovery-oriented, inquiry-based and problem-centered approaches□ Recognizes, formulates, and solves problems arising from mathematical situations, everyday experiences, applications to other disciplines, and career applications□ Identifies the steps in the problem-solving process, expresses these in a variety of forms, and monitors own progress toward solutions□ Summarizes results as a conclusion to a specific solution□ Applies results of a problem solution to other similar problems□ Employs deductive and inductive reasoning, formulating and testing conjectures, indirect proofs and a variety of example frameworks in the problem-solving process
(NS M5a,5b, 5c, 5d; NJ 4.1, NJ 4.2, NJ 4.3, NJ 4.4, NJ 4.5, NJ 4.6; NCTM; CK M1, CK M9)	

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Standard CH 3: 18	Benchmarks (High School)
<p>The student understands, selects, and applies various methods of performing numerical operations.</p> <p>(NS M6; NJ 4.2, NJ 4.5, NJ 4.8; NCTM)</p>	<ul style="list-style-type: none">□ Carries out all essential calculations, computations, and functions as deemed essential within the specific area of mathematical study identified□ Uses all basic and necessary terminology identified as appropriate to the specific area of mathematical study□ Creates for real purposes scales, diagrams, graphs, sketches, and written papers to express information and solutions□ Solves all types of equations deemed essential within the specific area of mathematical study□ Demonstrates an ability to use algorithms□ Demonstrates an ability over time to apply knowledge regarding basic solutions to solve more complex problems□ Uses calculators, computers, manipulatives and other mathematical tools (compasses, protractors, rulers, tapes) to enhance mathematical thinking, understanding, and problem solving

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High School Mathematics

<p>Standard CH 3: 19</p> <p>The student communicates mathematically through written, oral, symbolic, and visual forms of expression.</p> <p>(NS M8a, 8b; NJ 4.2, 4.16; NCTM; CK M9)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">□ Uses appropriate terminology, symbols, conventions, and visual aids to effectively communicate a mathematical concept, process, and solution□ Produces work which demonstrates proficiency (as to organization, clarity, accuracy) relative to all standards which are established to identify a certain area of study (e.g., algebra, geometry)□ Presents orally and in writing a variety of mathematical ideas, procedures, and solutions, including narrative accounts of simple and complex processes□ Writes succinct accounts of the mathematical results obtained in a project completed over time, with all appropriate visuals□ Reads a book written about the application of a mathematical principle, concept, or topic to a real-life situation, and reports on this book
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High School Mathematics

<p>Standard CH 3: 20</p> <p>The student applies mathematical knowledge and skills to relate to real-world situations.</p> <p>(NS M8; NJ 4.16; NCTM; CK M9)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">❑ Works at rich, open-ended problems which require the use of mathematics in meaningful ways❑ Conducts research relative to a real-world issue (health, poverty, discrimination, political, conflict, etc.), and uses logic, mathematical reasoning, and statistical strategies to generate a written report which explains the scope of the work, the value in studying the data, possible conclusions, and approaches to the issue❑ Conducts research relative to a physical phenomenon (physical system, physical structure, geographical configuration), and uses mathematical knowledge, models, and all other appropriate tools to generate a written report which explains the scope of the work, the value in studying the data, possible conclusions and approaches to the full understanding of the phenomenon❑ Creates a design for a physical structure using all appropriate mathematical ideas, techniques, and logic, and generates a report that explains in a variety of ways the purpose of the project, steps used in the creation of the structure, and the organizational plan❑ Applies mathematical knowledge and skill in presenting oral and written presentations which pertain to various real-world situations in business, political, and other systems (educational, medical, media)❑ Researches and prepares a report on the history of a mathematical idea
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High School Mathematics

<p>Standard CH 3: 21</p> <p>The student presents evidence that demonstrates understanding of differential and integral calculus.</p> <p>(NJ 4.15, NJ 4.16; NCTM)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">□ Graphs functions and uses calculus techniques to analyze and predict observed local and global behavior of functions□ Determines limits of functions intuitively, calculates limits using algebra, and estimates limits from graphs or tables of values□ Describes asymptotic behavior in terms of limits involving infinity, and compares relative magnitudes of functions and their rates of change□ Understands continuity intuitively and in terms of limits, including a geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem)□ Examines and describes the derivative geometrically, numerically, and analytically; interprets it as an instantaneous rate of change, and defines it as a limit of the difference quotient□ Determines the slope of a curve at a point, and finds the equation for a tangent line to the curve at that point using local linear approximation and techniques of differentiation <p>*Continued on next page*</p>
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High School Mathematics

<p>Standard CH 3: 21 (Continued)</p> <p>The student presents evidence that demonstrates understanding of differential and integral calculus.</p> <p>(NJ 4.15, NJ 4.16; NCTM)</p>	<p>Benchmarks (High School)</p> <ul style="list-style-type: none">□ Solves equations involving derivatives including rates of change, velocity, acceleration, and optimization problems□ Uses implicit differentiation to find the derivative of an inverse function□ Computes Reimann sums using left, right, and midpoint evaluation points, and expresses the definite integral as a limit of Reimann sums over equal subdivisions□ Understands and uses the basic properties of the definite and indefinite integral in a variety of applications to model physical, social, and economic situations□ Uses the Fundamental Theorem of Calculus to evaluate definite integrals and to represent a particular antiderivative□ Applies the definite integral to find the area of a region, the volume of a solid with known cross sections, the distance traveled by a particle along a line, volumes of solids of revolution, arc length, and area of surface of revolution□ Uses techniques of antidifferentiation including substitution, trigonometric substitution, integration by parts, and partial fraction decomposition□ Solves separable differential equations and uses them in modeling
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Resources

Marzano, R.J., and Kendall, J.S. (1997) *Content Knowledge, A Compendium of Standards and Benchmarks for K-12 Education.* Colorado: Mid-continent Regional Education Laboratory and Alexandria, Virginia: Association for Supervision and Curriculum Development. *

National Council of Teachers of Mathematics. (2000) *Standards for Mathematics.* Urbana, Illinois: National Council of Teachers of Mathematics.

New Jersey Department of Education. (1996) *Core Content Standards.* Trenton, New Jersey.

New Standards. (1997c) *Performance Standards: English Language Arts, Mathematics, Science, Applied Learning;* Volumes 1, 2,3. Washington, DC: National Center on Education and the Economy. **

* The information contained in this resource represents seven years of work on the part of many who labored on the McRel Project. Its purpose was to survey and consolidate the many national and state-level efforts to identify what K-12 students should know and be able to do in a variety of areas. To accomplish this goal, McRel researchers consulted 116 national and state-level documents that address standards and benchmarks in various domains. The effort

resulted in 255 standards with their accompanying 3,968 benchmarks, which are detailed in fourteen different categories, one of which is Mathematics.

** The New Standards document is the result of a collaboration of the Learning Research and Development Center of the University of Pittsburgh and the National Center on Education and the Economy, in partnership with states and school districts. After researching and evaluating many national and state-level documents, the “New Standards” organization shared a Consultation Draft with researchers and educators in nineteen other countries, asking that the document be reviewed in terms of their own country’s standards, and in light of what is considered world class in their field.